SEASON 1

An item at a department storeis priced at 189.99 and can be bought by making 20 payments of 10.50. Find the interest rate, assuming that interest is compounded monthly.

There are 10 cookies to be split among 5 kids.
Any kid could get as many as 10 cookies or none at all.
However, these cookies are made of titanium, and thus not able to be broken apart.
Considering each child as distinct individuals, how many ways could the distribution happen?
(Do not worry about the children's teeth breaking apart from the titanium, it's a math problem, and in these, nobody cares if you eat 235 candy bars or buy 61 watermelons at a store. Logic is thrown out the window. As a competitive math student, you should have known that by now. Shame on you.)

Consider the following polynomial:

$$x^4 + 7x^3 + 3x^2 + 4x + 2$$

Let r_1, r_2, r_3, r_4 be its roots. Find the value of the following expression:

$$\frac{r_1^2 r_2 r_3 r_4 + r_1 r_2^2 r_3 r_4 + r_1 r_2 r_3^2 r_4 + r_1 r_2 r_3 r_4^2}{r_1 r_2 + r_1 r_3 + r_1 r_4 + r_2 r_3 + r_2 r_4 + r_3 r_4}$$

There exists a cyclic quadrilateral with vertices labeled A, B, C, D. We have that $AB = 8$, $AC = 12$, $BD = 15$, and $CD = 10$. Find $AD \cdot BC$.

Find the value of $1^3 + 2^3 + 3^3 + 4^3 + 5^3 + 6^3 .... + 197^3 + 198^3 + 199^3 + 200^3$.

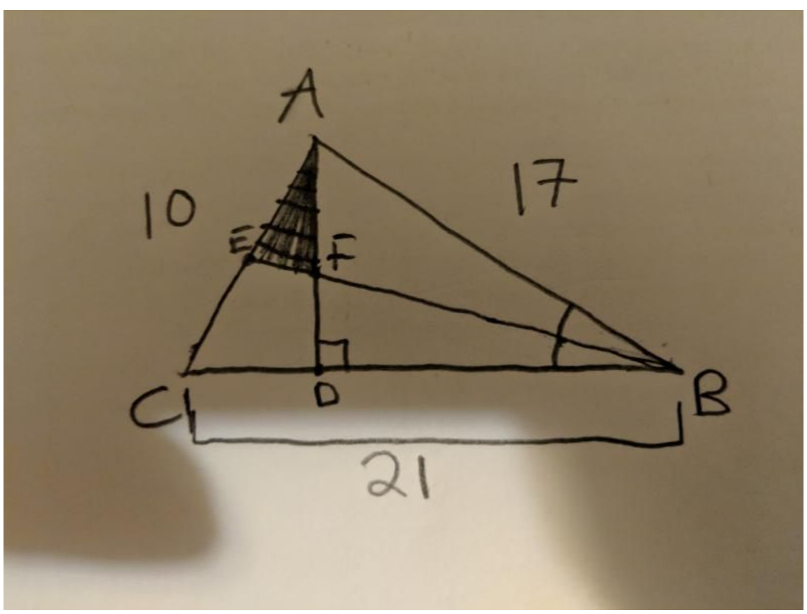
Please do not use a calculator.

There are multiple patterns you can find to help you solve this.

In the year 2001, the United States will host the the International Christmas Gala. Let $I, C,$ and G be distinct positive integers such that the product of all three integers equals 2001. What is the largest possible value of the sum $I + C + G$?

Suppose x, y, z are each chosen randomly from the interval 0 to 1. What is the probability that they form the side lengths of a triangle with a perimeter of less than 2?

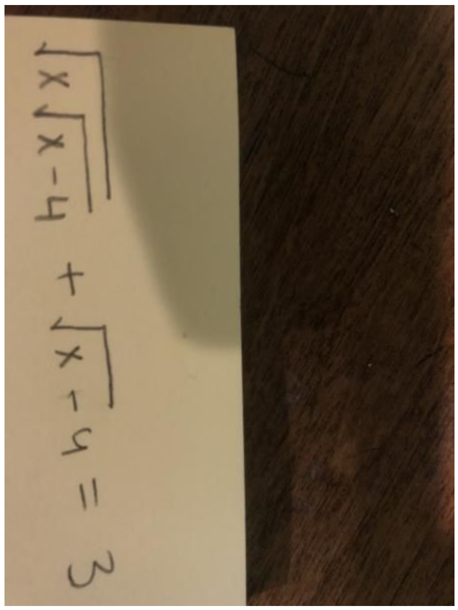
Suppose $\triangle ABC$ has side lengths $AB = 17, BC = 21, AC = 10$. Let D be the foot of the altitude from A . Furthermore, suppose the angle bisector of $\angle ABC$ intersects AC at E and AD at F . The area of triangle AFE can be expressed as $\frac{m}{n}$ where m and n are relatively prime; i.e., the fraction is in simplest form. Compute $m + n$.



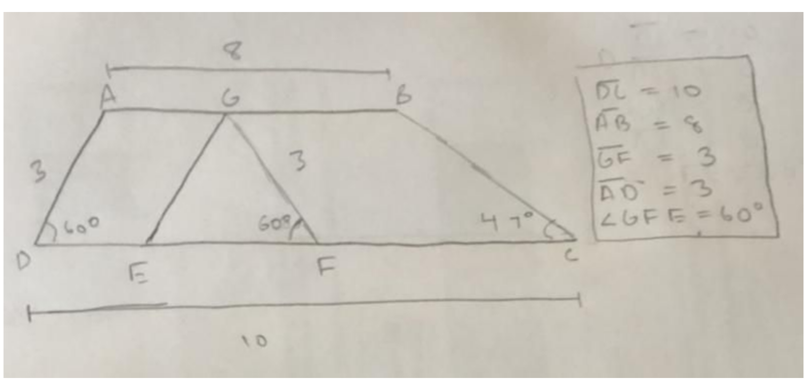
Find the number of unordered quadruples of non-negative integers (a, b, c, d) such that they satisfy $a + b + c + d = 18$.

@everyone

Manipulate the below equation so that one side of the equation is -5. On the other side, remove all square roots and fully factor the equation. (sorry for the poor wording kinda low on time today)



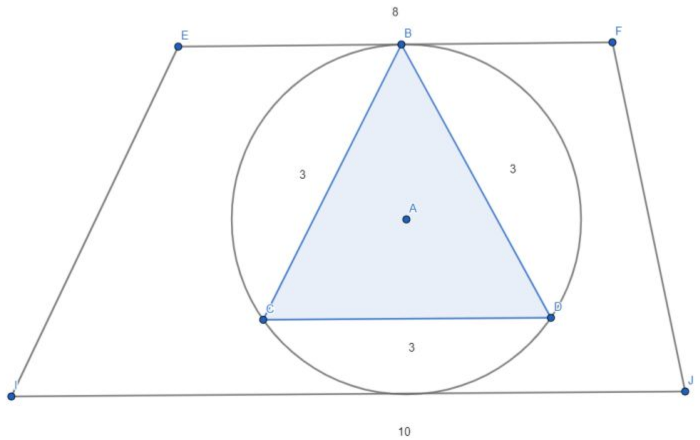
Find the area of the trapezoid in the diagram below. DC= 10 AB= 8 GF= 3 AD=3 and Angle GFE = 60 Degrees Angle ADE= 60 Degrees.



Integers x and y with $x > y > 0$ satisfy $x + y + xy = 624$. Find x.

Determine the value of n that maximizes $\frac{205^n + 203^n}{n!}$ over the positive integers.

Find the coefficient of the x^5y term in the expression $(2x+3y)^6$.



FIND THE AREA OF ABOVE

The polynomial $f(x) = 2x^3 + px^2 + 2x + r = 0$ has the property that the average of its zeros and the product of its zeros are equal. The y-interecept of graph $y = f(x)$ is 10. Find the sum of the coefficients of x , x^2 , and x^3 .

Find lowest possible sum where coefficients are integrers.

What is the probability that a randomly chosen palindrome between 1,000 and 10,000 is divisible by 13? A palindrome is a number that reads the same way forwards and backwards. For example, 101, 919, 1661, 5335, and 9779 are all palindromes.

Find the sum of all possible values of x in the expression $|2x-8|+6 = |6-3x|+19$

Suppose a, b, c, d, e, f are real numbers that add up to 10 and $(a-1)^2 + (b-1)^2 + (c-1)^2 + (d-1)^2 + (e-1)^2 + (f-1)^2 = 6$. What is the maximum possible value of f ? Express your answer as a common fraction in simplest form.

Let $\kappa(n)$ denote the sum of the digits of n , find $\kappa(1)+\kappa(2)+\kappa(3)+\cdots+\kappa(4036)+\kappa(4037)+\kappa(4038)$

Find the area of all rectangles with integer side lengths x and y whose area is equal to its perimeter.

You have 8 balls and split them into two groups of 4. You then color all 8 balls with four different colors randomly and independently of each other. The probability that no two balls from different groups will have the same color is $\frac{m}{n}$ where it is in simplest form. Determine m .

Find all integer triples a, b, c that satisfies the equation $\frac{21}{16}=\frac{1}{a}+\frac{1}{ab}+\frac{1}{abc}$,where the average of a, b, and c is 3.

There are exactly 5 cars labeled $A, B, C, D,$ and E and there are also 5 parking spaces labeled $A, B, C, D,$ and E . The 5 cars go inside the parking lot and their parking spaces are randomly chosen. How many possible arrangements of cars can you have where there is at least 1 car on a parking space with the same letter?

Find the last two digits of a_{676} where $a_n=(a_{n-1})^2+25(a_{n-2})$ and $a_1=1, a_2=2$

SEASON 2

Determine the probability that $\lfloor\log_2 a\rfloor=\lfloor\log_2 b\rfloor$ where a and b are chosen randomly from the interval $(0,2)$.

Suppose ABC is a triangle where $AB=13, BC=5, AC=12$. Suppose the circle that is tangent to AC , BC , and the circumcircle (circle that goes through A, B, C) of $\triangle ABC$ has center O . What is the area of triangle OBC ?

Consider a $5x5$ grid with the bottom left corner denoted by the point $(0,0)$ and the top right corner denoted by the point $(5,5)$. How many paths are there from the bottom left corner to the top right corner with the condition that you cannot pass through the point $(4,1)$?

Tejas, Ayush, Saiesh, Dhruv, Hari, and CardiacPack have internet accounts. Some, but not all, of them are internet friends with each other, and none of them has an internet friend outside this group. Each of them has the same number of internet friends. In how many different ways can this happen ?

Awesome Bowl 53 is expected to seat far more people than usual. They have arranged seating for 57744 people. However, between costs of seats and expected concessions, they must fill all seats. Furthermore, Goger Roodell has stated that all parties must have the same number of people, and that the number of people in the first party to buy the ticket must be the number of people in each party. I never did figure out why he makes such dumb rules... Anyway, assuming that Awesome Bowl 53 takes place between the Sheep and Colonialists, what is the sum of all possible number of parties attending the game? Please do not use a calculator. We would encourage a solution be posted with it, but it is not required .

Let $x < y < z$ be three integers such that x, y, z is an arithmetic progression and x, z, y is a geometric progression. What is the smallest possible value of z ?

Find the number of 15 letter strings where each letter is either X or Y and there are no more than 3 X s or 3 Y s in a row.

Determine the product of all integers x such that $\log_2(x^2 - 4x - 1)$ is also an integer.

Suppose you have 16 identical balls. One of them is slightly heavier than the rest, but you can't tell which one it is. You are given a balance scale. How few weighings will it take to discern which ball is the slightly heavier?

The graph of $y = x^2$ is rotated $45°$ counterclockwise about the origin. The line $x = -3\sqrt{2}$ intersects the rotated graph at two points. Determine the sum of the y-coordinates of these two points. Express your answer in simplest radical form.

SAMOSA PROBLEM - ONLY OGS KNOW

Hari is collecting different types of samosas to do math problems with. At the end of each day, Hari eats ONE random type of samosa out of N random types. He may eat a samosa of a type he has already eaten. Sushrit has calculated that Hari, on his way to eat every samosa there is, will tbe eating for about 40 days.(This is a rough estimate). If Hari has an arbitrarily large but equal number of each type of samosa (his eating doesn't deplete the pool of samosas), how many different types of samosa does he own?

Suppose ABC is a right triangle in which AB is the hypotenuse and has length 24 . Squares $ABMN$ and $ACXY$ are constructed exterior of the triangle. Suppose M, N, X, Y lie on a circle. Compute the area of triangle ABC .

Compute the number of ordered pairs of integers (a, b) such that the polynomials $x^2 - ax + 24$ and $x^2 - bx + 36$ have one root in common.

Let S_k , where $k \in \{1, 2, \ldots, 99\}$ denote the sum of the infinite geometric series whose first term is $\dfrac{k-1}{k!}$ and with common ratio $\dfrac{1}{k}$. Find the value of $\dfrac{100^2}{100!} + \sum_{k=1}^{99} |(k - 1 - \frac{1}{k})S_k|$

Suppose ABC is an equilateral triangle with side length 2. A sphere with radius 1 is drawn at each of the vertices of the triangle. All of these 3 spheres are internally tangent to another larger sphere. Finally, a sphere that is internally tangent to the larger sphere and externally tangent to each of the original 3 spheres is drawn. The radius of this sphere can be written as $\dfrac{a+\sqrt{b}}{c}$ where a, b, c are integers and b is not divisible by the square of any prime. Compute $a + b + c$.

Let a, b, c be the roots of the cubic $x^3 - 2x^2 + 4x - 7$. Find $\left(\dfrac{1}{a+b} + \dfrac{1}{b+c} + \dfrac{1}{c+a}\right)^2$.

Due to the no-call of the pass interference made by Goger Roodell on Bickel Bobey-Boleman, the Colonialists and Sheep will play tomorrow in Awesome Bowl 53 with a total of 57,744 people watching. (If you get the joke, I salute you.)

1. In Awesome Bowl 53, Colonialists QB Bom Trady throws 7 interceptions, and the Sheep win 69-17 with no defensive points. Assume that all TDs are worth 7 points, and all FGs are worth 3 points, and that a TD and FG is the only way of scoring. Find the least amount possible offensive drives (scoring opportunities) the Sheep got.
2. After the game, President Unnamed Fellow Because Akshar Says No Politics makes a speech on Unnamed Topic Because Akshar Says No Politics. President Unnamed Fellow Because Akshar Says No Politics speaks at a very fast rate of 6 words per second, but it is such a long speech that it takes President Unnamed Fellow Because Akshar Says No Politics 42 minutes to say it. President Unnamed Fellow Because Akshar Says No Politics also insists that all pages must have the same number of words on them. Find the amount of distinct possible numbers of pages.
3. A week after the game, Bom Trady and the Colonialists sue the NFL (National Failed-Call League) for allowing them to lose. Somehow, the Colonialists win the lawsuit, and Roodell agrees that the Awesome Bowl 53 should be split up among all 53 players on the roster, giving each of them an Awesome Bowl 1 trophy. In 1967, since the NFL was short on money, they only gave out 27 Awesome Bowl trophies. Noting that the value of the trophy is inversely proportional to its abundance, find the percent loss of the value of an Awesome Bowl 1 trophy after the lawsuit. Round it to the nearest whole number. NO. You may not use a calculator.

Once you get the three answers, divide the first answer you got by the second answer you got, then multiply by your third answer.

Compute the minimum value of $\sqrt{a^2+9} + \sqrt{(b-a)^2+1} + \sqrt{(c-b)^2+4} + \sqrt{(10-c)^2+1}$ where a, b, c, d are real numbers.

A fixed point P is given inside a sphere with center O . Three mutually perpendicular rays from P intersect the sphere at points $U, V,$ and W . point Q is the vertex diagonally opposite to P in the parallelepiped with edges PU, PV and PW . Show that OQ is the same for all possible choices of the three mutually perpendicular rays from P .

Say Q is the
vertex diagonally opposite to P in the parallelepiped determined by PU, PV,

and PW. Find the locus of Q for all sets of 3 r rays from P.

Find the ratio of the number of even divisors to the number of odd divisors of $2019!$. Express your answer as $a : b$, where a, b are positive integers.

Find the smallest positive integer n such that both $n + 2002$ and $n - 2002$ are perfect squares.

Let $f(x)$ equal the number of zeroes to the right of the rightmost non-zero digit in the decimal form $x!$, and let $n = \dfrac{5^{2008} + 2}{3}$. Given that $f(n)$ can be written as $\dfrac{5^a - b}{c}$, where b and c are relatively prime positive integers, b is less than 10^5 , and c is less than 10^2 , find $a + b + c$.

Let $f_1(x) = 1 + \dfrac{1}{x}$ and $f_n(x) = 1 + \dfrac{1}{f_{n-1}(x)}$ for $n \geq 2$. Compute the value of x that satisfies $f_{12}(x) = 0$. Express your answer as a common fraction in simplest form.

Consider the sequence $A(n)$ equals $\sin(n)$. Of course, n varies among the positive integers and is measured in radians. Prove that for any integer X , there is another integer Y such that $A(Y) > A(X)$

Determine the remainder of the coefficient of x^2 in the expansion of $(1+x+x^2)(1+2x+2x^2)(1+3x+3x^2)\cdots(1+19x+19x^2)(1+20x+20x^2)$ when divided by 1000 .

Determine the remainder of the coefficient of x^2 in the expansion of $(1+x+x^2)(1+2x+2x^2)(1+3x+3x^2)\cdots(1+19x+19x^2)(1+20x+20x^2)$ when divided by 1000 .

A square has 3 vertices with x-coordinates of $0, 2, 18$. Determine the sum of all possible areas of the square.

3 circles with radii 3 , 4 , and 5 respectively are drawn mutually tangent to one another. Jim draws a fourth circle which is tangent to all three other circles, meaning the four are mutually tangent. What is the radius of the circle Jim drew if he drew the largest possible such circle?

Let $d(n)$ be the number of positive divisors of n . Find the sum of all positive integers less than a 100 that satisfy $d(n) + d(n+1) = 9$.

How many five-digit positive integers are there such that the sum of their digits is equal to the number formed by the first two digits?

Consider all 10 - digit base 2 numbers. A number is taken at random from this pool of numbers, and does not contain two consecutive 1s with probability $\frac{m}{n}$. Find $m + n$.

The sequence $(a_n)_{n=0}^{\infty}$ is defined by $a_0 = \frac{1}{2}$ and $a_n = 1 + (a_{n-1} - 1)^2$. Compute $a_0 a_1 a_2 \cdots$.

Triangle ABC has side lengths $AB = 13, BC = 15, AC = 14$. What is the probability that a randomly chosen point inside the triangle will be closer to A than both B and C ? Express your answer as a common fraction in simplest form.

Let S be the set of divisors of $67(9!) + 27(8!)$. Compute the median of S.

Suppose the roots of the polynomial $x^3 + 6x^2 + 4x + 2$ are r_1, r_2, r_3 . Also suppose that $x^3 + ax^2 + bx + c$ is the polynomial with roots $r_1 - 1, r_2 - 1, r_3 - 1$. Compute $a + b + c$.

In cyclic quadrilateral $MNOP$, angle $PON = 60°, NO = 13, OP = 7,$ and $PM = 3$. The area of the quadrilateral can be expressed in the form $\frac{a\sqrt{b} + c\sqrt{d}}{e}$, where a, b, c, d, e are positive integers, and b, d are distinct and have no perfect square factors. What is $a + b + c + d + e$?

Let n be an integer such that $n^2 + 6n + 24$ is a perfect square. Suppose that a is the least possible value of n , and b is the largest possible value of n . What is $b - a$?

$\triangle ABC$ is such that $AB = 13, BC = 14, CA = 15$. Let D be the foot of the altitude from A onto BC . Let E be the intersection of the common external tangent of $\triangle ABD$ and $\triangle ADC$ other than BC with AD . Compute AE .

Compute $\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2 n}{3^m (n \cdot 3^m + m \cdot 3^n)}$.

Let n be the number of ordered quadruples (x_1 , x_2 , x_3 , x_4) of positive even integers that satisfy $\sum_{i=1}^{4} x_i = 104$ Let a be $\frac{n}{25}$. Suppose that b equals the sum of the digits of a , and c equals the product of the digits of a . Find the sum of the numerator and denominator when $\frac{b}{c}$ is expressed in lowest terms.

Let $P(n)$ be the n th prime number. Prove that $P(1) \cdot P(2) \cdot P(3) \cdot \cdots \cdot P(n-1) \cdot P(n) + 1 + 3 + 5 + 7 + \cdots + (2n-3) + (2n-1)$ is never a perfect square.

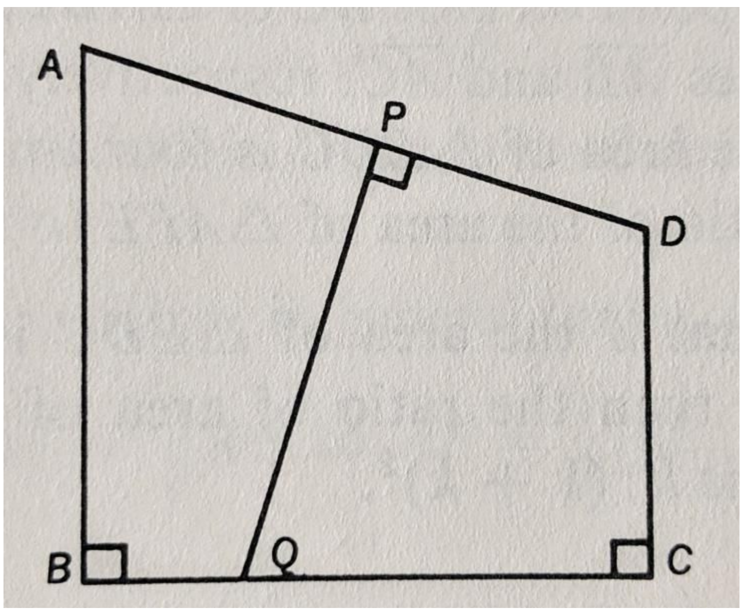
Starting at $(0,0)$, a turtle moves in the coordinate plane via a sequence of steps, each of length one. Each step is left, right, up, or down, all four equally likely. Find the probability that the turtle reaches $(2,2)$ in six or fewer steps.

Determine the number of numbers in $1, 2, 3, \cdots, 2017$ that don't divide $\frac{\operatorname{lcm}(1,2,3,\cdots,2017)}{2016}$, where lcm denotes the least common multiple.

10 points are evenly spaced around a circle with radius 1. How many non-congruent triangles can be made by selecting 3 of the 10 points as vertices?

SEASON 3

In the following diagram, right angles are indicated and $AB = 9, BC = 8, CD = 7$ and P is the midpoint of AD. Compute the area of quadrilateral $APQB$.



Today is both Kenan and Noah's 15th birthday!
Given that there are 56 members of CNCM besides them two, what is the probability that at least 2 other people share a birthday? Round it to the nearest thousandth of a percent. You may use a calculator, but I would like a solution.
Assume that nobody was born on February 29th.

Suppose a, b, c are the roots of $x^3 - 2x^2 + 3x - 4 = 0$. Compute $\frac{1}{a(b^2+c^2-a^2)} + \frac{1}{b(c^2+a^2-b^2)} + \frac{1}{c(a^2+b^2-c^2)}$.

Call a number "high iq" if it is divisible by the sum if its digits in base 23. How many prime "high iq" numbers are there on the interval $[\ 2_{23}, 1K8_{23}\ ]$?

Suppose $x_1, x_2, x_3, \cdots, x_{2017}$ are the distinct complex numbers that satisfy the equation $x^{2017} = x + 1$. Find $\sum_{n=1}^{2017} \frac{x_n}{x_n^2 + 1}$.

Determine the value of $\sum_{i=0}^{25} \binom{25-i}{i}$.

What is that largest positive integer n for which $n^3 + 100$ is divisible by $n + 10$?

Noah and Akshar are playing a game of ping-pong.

Ping-pong is played in a series of consecutive matches, where the winner of a match is awarded one point.

In the scoring system that Noah and Akshar use, if one person reaches 11 points before the other person can reach 10 points, then the person who reached 11 points wins. If instead the score ends up being tied 10-to-10, then the game will continue indefinitely until one person's score is two more than the other person's score, at which point the person with the higher score wins. (You need to reach 11, but you must win by at least 2.)

The probability that Akshar wins any one match is 70% and the score is currently at 9-to-9. What is the probability that Noah wins the game?

Express your answer as a common fraction.

Find the sum of the digits of $5\sum_{k=1}^{99} k(k+1)(k^2+k+1)$

Find the number of subsets of $1, 2, 3 ..., 16, 17$ such that the sum of the elements of each such subsets is greater than 76. Do not include the whole set in your answer (that is, the subset 1,2,3...17). Your answer should be an integer.

Each subset

Define $T_n = \dfrac{n(n+1)}{2}$ and $S_n = n^2$. Compute

$$\sqrt{S_{62} + T_{63}\sqrt{S_{61} + T_{62}\sqrt{\cdots\sqrt{S_2 + T_3\sqrt{S_1 + T_2}}}}}.$$

Compute the sum of all positive integers such that $\tau(n)^2 = 2n$ where $\tau(n)$ is the number of divisors of n .

Find the number of ordered triples (a, b, c) where a , b , and c are positive integers, a is a factor of b , a is a factor of c , and $a + b + c = 100$.

How many positive integers less than $10,000$ have at most two different digits?

Compute $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \cdots + \frac{1}{\sqrt{2024}+\sqrt{2025}}$.

There exists a fourth degree polynomial such that $f(3) = 103$, $f(4) = 12$, $f(5) = 9$, $f(6) = 12$, and $f(7) = 103$. Define $g(x)$ as $f(2x+4)$. Find the coefficient of the x^4 term of $g(x)$.

Let ABC be a triangle with $AB = 10$, $AC = 11$, and circumradius 6 . Points D and E are located on the circumcircle of $\triangle ABC$ such that $\triangle ADE$ is equilateral. Line segments $\overline{DE}$ and $\overline{BC}$ intersect at X . $\frac{BX}{XC}$ can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

There are 53 days until CNCM Math Bowl!

Akshar has 53 fair coins.

He takes the time, magically not getting unbelievably bored, and flips each coin individually.

What is the probability that at least 30 coins either show heads or at least 30 coins show tails?

Express your answer as a fraction.

If you need to include an exponent in your answer, that is fine, as long as it is clear what you are saying.

Let n be the number of 100-digit numbers that are divisible by 3 and only use the digits $2, 4, 6, 9$. They don't necessarily have to use all those digits. n can be expressed as $\dfrac{2^a + b}{c}$ where c is odd and $b < c$. Compute $a + b + c$.

Suppose a is the largest integer such that a^2 can be expressed as a difference of cubes, $2a+79$ is a perfect square, and $a < 1000$. Let r be the remainder when a is divided by 13, and let I equal the sum of the coefficients of $p(x) = (x-r)(x+r+1)(x-(r-2))$. Find the largest number x such that $x^2 < I$.

In triangle ABC , $\tan \angle CAB = \frac{22}{7}$, and the altitude from A divides BC into segments of length 3 and 17 . What is the area of triangle ABC ?

Find the sum of all positive integers n such that $(n-1)^3$ divides $n^{2018(n-1)} - 1$.

Find the number of UNordered pairs of integers (x, y) such that $x^2 + 4xy + y^2 = 21$. Note that (x, y) is taken to be equivalent to (y, x)

Define the sequence $\{a_n\}$ by $a_{n+2} = a_n + 7a_{n+1}^2$ where $a_1 = 2$ and $a_2 = 50$. Find the remainder when $a_{2019!}$ is divided by 41.

There exist two unique nonzero integers a, b such that the polynomial $x^3 + ax^2 + bx + 9a$ has 3 integer roots, two of which are the same. Compute $a+b$.

Trapezoid $MATH$ has $MA = AT = TH = 5$ and $MH = 11$. Let S be the orthocenter (intersection of altitudes) of $\triangle ATH$. Compute the area of quadrilateral $MASH$.

Suppose we have $\triangle ABC$ such that $\overline{AB} = 5$, $\overline{BC} = 7$, and $\overline{AC} = 8$. Let D be a point on $\overline{BC}$, and E and F be the incenters of $\triangle ABD$ and $\triangle ACD$. The circumcircles of $\triangle BED$ and $\triangle CFD$ meet at distinct points p and q. The maximum possible area of $\triangle BPC$ is $a - b\sqrt{c}$, where a and b are positive and c is not divisible by the square of any prime. Find $\frac{a+b+c}{2}$.

When an unfair coin is flipped five times, the probability of getting heads exactly once is the same as that of getting heads twice and the probability is $\neq 0$. Find the probability of flipping exactly 3 heads out of 5 flips using this unfair coin.

Define the function $f(a,b,c,d,e) = \dfrac{a^2+b^2+c^2+d^2+e^2}{cde+abc}$ for $a, b, c, d, e > 0$. Find the minimum of f . Express your answer in simplest radical form.

Define the function $f(a,b,c,d,e) = \dfrac{a^2+b^2+c^2+d^2+e^2}{cde+abc}$ for $a, b, c, d, e > 0$. Find the minimum of f . Express your answer in simplest radical form.

An equilateral triangle in Quadrant 1 has points $(0, 0)$, $(x_1, 4)$, and $(x_2, 11)$.
A) How many of the variables out of (x_1, x_2) can be determined?
B) Find the sum of all variables out of (x_1, x_2) that are determinable. Make sure you make it cleae which answer corresponds to each part

Find the positive integer n such that $n!$ has $n - 48$ trailing zeros.

SEASON 4

$\sin 2° \sin 4° \sin 6° \cdots \sin 90°$ equals $\frac{p\sqrt{5}}{2^{50}}$ where p is an integer. Compute p .

How many positive integers have exactly three proper divisors (positive integral divisors excluding itself), each of which is less than 50 ?

Let K be the product of all factors $(b - a)$ (not necessarily distinct) where a and b are integers satisfying $1 \le a < b \le 20$. Find the greatest positive integer n such that 2^n divides K .

Jerav is considering consecutive positive integers. He is trying to find all sets of 6 consecutive integers { x_1 , x_2 .... x_6 } such that the integers x_n (not necessarily ordered in any meaningful way) satisfy $x_1\ x_2 + x_3\ x_4 = x_5\ x_6$ for at least one ordering of the set. However, Jarav has to grind his spanish theme song project over spring break and thus now you have tk do it. GL!

A positive integer N is such that if you concatenate two copies of N , then you get a perfect square. For example, concatenating two copies of 123 would result in the number 123123 . Find the remainder when the smallest such N is divided by 1000 .

Suppose $ABCDEFG$ is a regular heptagon inscribed in the unit circle with center O . Let point P be such that OAP is an isosceles right triangle with a right angle at A . Compute the value of $(AP \cdot BP \cdot CP \cdot DP \cdot EP \cdot FP \cdot GP)^2$.

Let $S = \{2^0, 2^1, 2^2, \ldots, 2^{10}\}$. Consider all possible positive differences of pairs of elements of S . Let N be the sum of all of these differences. Find the remainder when N is divided by 1000 .

Compute $\sum_{a=1}^{\infty}\sum_{b=1}^{\infty}\sum_{c=1}^{\infty} \frac{ab(3a+c)}{3^{a+b+c}(a+b)(b+c)(c+a)}$. Express your answer as a common fraction in simplest form.

Find all ordered triples (a, b, c) such that $\frac{(a-b)(b-c)(c-a)}{2} + 2$ is of the form 2016^n with n being a nonnegative integer.

A number N has three digits when expressed in base 7 . When N is expressed in base 9 the digits are reversed. What is N when expressed in base 10 ?

A square has sides of length 4 . Set S is the set of all line segments that have length 4 and whose endpoints are on adjacent sides of the square. The midpoints of the line segments in set S enclose a region whose area can be expressed as $a - b\pi$, where a and b are nonnegative real numbers. Find $a + b$

A fair coin is to be tossed 10 times. Find the probability that heads doesn't occur on consecutive tosses.

In triangle ABC angle C is a right angle and the altitude from C meets $\overline{AB}$ at D. The lengths of the sides of $\triangle ABC$ are integers, $BD = 29^3$. Find $\cos B$. Express your answer as a common fraction in simplest form.

$x^3 + y^3 + (x+y)^3 + 30xy = 2000$ for x, y positive reals. Find the average of all possible values of $x + y$.

Find the sum of all possible values of $a + b$ given that a and b are positive integers such that $\operatorname{lcm}(a, b) - \gcd(a, b) = 103$.

Possible season 5

Define the sequence a_n by $a_0 = 1$ and $a_{n+1} = a_n^2 + 1$ for nonnegative integers n . Compute $\gcd(a_{1999}, a_{2004})$.

Convert $8.\overline{72}_9$ as a base 10 number. Please express your answer as a common fraction.

Suppose $\triangle ABC$ has area 5 and $BC = 10$. Let F and E be the midpoints of sides AB and AC , respectively. Let G be the intersection of BE and FC . Given that quadrilateral $AFGE$ can be inscribed in a circle, what is the value of $AB^2 + AC^2$?

There exists a quadrilateral $ABCD$ whose diagonals are AC and BD . If $\angle ADB$ is 80 degrees, $\angle ACB$ is 40 degrees, and $\angle BAC$ is 30 degrees, find the measure of $\angle BDC$

A quadrilateral is inscribed in a circle of radius $\sqrt{32}$. Three of the sides have length 4 . Find all possible areas of the quadrilateral.

Given that $4\cos x - 3\sin x = \frac{13}{3}$, what is the value of $7\cos(2x) - 24\sin(2x)$? Express your answer as a common fraction in simplest form.

Unable to retrieve this problem rip

A regular decagon is inscribed in a circle, and another regular decagon is circumscribed about the same circle. What is the ratio of the length of the side length of the smaller decagon to the larger decagon? Leave your answer to the nearest hundreth .

Let S be a set of 5 elements. In how many different ways can one select two not necessarily distinct subsets of S so that the union of the two subsets is S ? Selection order doesn't matter.

Tim and Tom play a game with a magical 3 -sided coin with sides Heads, Tails, and Middles, where each side has an equal probability of being landed on. Tim and Tom each take turns playing this game. A turn consists of flipping three coins one at a time, and a round consists of Tim's turn and Tom's turn. A winner is declared if they flip Heads, Tails, or Middles in any order (for example, [Heads, Middles, Tails] and [Middles, Tails, Heads] win the game, while [Tails, Tails, Heads] or [Middles, Middles, Middles] do not win the game). The probability that Tom wins the game on any round after the 1st round (if Tim goes first) can be expressed as $\frac{m}{n}$ where m and n are relatively prime. Find $m+n$.

The numbers a , b , c are the distinct roots of the polynomial $P(x)=x^3-10x^2+x-2015$. The cubic polynomial $Q(x)$ has a leading coefficient of 1 and has distinct roots $bc-a^2$, $ca-b^2$, $ab-c^2$. What is the sum of the coefficients of Q ?

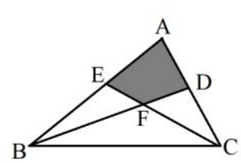
There exists 3 numbers $1515, 1718, 2019$ such that if the number x is added to all 3 numbers, the resulting numbers are the squares of an arithmetic sequence. Find x.

12 distinct points are placed on a circle. How many distinct polygons can be created with these points? (Polygons are closed figures with 3 or more straight sides)

Find the number of ordered pairs of integers (a, b, c) such that $\frac{1}{a}+\frac{1}{b}=\frac{1}{c^2}$ where c is either 2012 or 2013.

Suppose square $ABCD$ has side length 100 and M is the midpoint of side AB . Furthermore, let P be a point in square $ABCD$ such that $MP=50$ and $PC=100$. Compute AP^2 .

Suppose $\triangle ABC$ has an area of 40, and is divided into three smaller triangles and one quadrilateral. If $\frac{AD}{DC}=\frac{2}{3}$ and $AE=BE$, find the area of the shaded region.



POTD TIME

For how many positive integers R less than 131 is $R!$ divisible by $R+1$?

Subset A contains all odd numbers less than 100 , and Subset B contains all positive triangular numbers less than 100 . How many subsets of A do not contain any elements from the intersection of A and B ?

SEASON 5

Calculate $\sum_{k=1}^{\infty} \tan^{-1}\left(\frac{1}{k^2-k+1}\right)$.

Find the sum of all 6 -digit palindromes.

Suppose we have the three vectors in $\mathbb{R}^3$: $\boldsymbol{a_1} = <1,-2,3>$, $\boldsymbol{a_2} = <5,-13,-3>$, and $\boldsymbol{b} = <-3,8,x>$. Define P as the plane through the origin in $\mathbb{R}^3$ contaning all scalar multiples of $\boldsymbol{a_1}$ and $\boldsymbol{a_2}$. Find all values of x such that $\boldsymbol{b}$ lies in P .

If 4 points are placed on the surface of a sphere, what is the probability that they can also be placed on the surface of a hemisphere?

Compute the area of the region containing all points (x, y) that satisfy $\begin{cases} y \le 15 \\ y \ge (2x+5)\left(\frac{|x|}{x}\right) \end{cases}$.

Find the smallest integer x such that $1^2 + 2^2 ... + x^2$ is a multiple of 69

How many positive five-digit integers are such that the product of their digits is either 599 , 600 ,or 601 ?

Triangle ABC has side lengths $4\sqrt{2}, 7, 5$. What is the area of the triangle whose side lengths are the sines of the angles of triangle ABC ?

Let x and y be positive reals such that $\log_{27}(x^6) + \log_3 y = \log_3 x + \log_{81}(y^8) = 6$. Find x^2y^2 .

For triangle ABC , points D and E are on sides AB and AC respectively so that $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{3}$. Triangle ADE is shaded, and the intersection of DC and EB is called X . The same procedure is performed on ABC will be performed on XBC , and all following triangle infinitely. What is the ratio of the shaded to unshaded area?

Quadrilateral $ABCD$ has side lengths $AB = 7$, $BC = 7$, $CD = 5$, $DA = 3$ and angle ABC is 60° . What is the length of diagonal BD ?

There are 5 people who each flip a coin. Everybody who flipped a heads will then proceed to flip their coin again. What is the probability that in the end all 5 will have a tails? Express your answer as a common fraction in simplest form.

Find the largest integer n such that $2^n + 2^8 + 2^{11}$ is a perfect square.

$\sum_{k=1}^{50} k\binom{50}{k} = a * 2^b$ when a is an odd number. Find $a + b$.

Suppose x is a real number that satisfies $\frac{(1+x)^2}{1+x^2} = \frac{13}{37}$. Compute $\frac{(1+x)^3}{1+x^3}$. Express your answer as a common fraction in simplest form.

There is a complex number z with imaginary part 164 and a positive integer N such that $\frac{z}{z+N} = 4i$. Find N

Define $f(x) = \frac{ax+b}{cx+d}$, for nonzero numbers a, b, c, d , that has the properties $f(17) = 17$, $f(56) = 56$, and $f(f(x)) = x$ for all values of x except for $\frac{-d}{c}$. Find the sum of the values that are not in the range of f .

An isosceles trapezoid has parallel sides of length 15 and 60. A circle is inscribed in the trapezoid, touching all four sides. What is the radius of the circle?

Circle O has radius 12 and point P is 8 units away from O . The midpoints of every chord that goes through P creates a closed figure. Find the sum of its area and perimeter.

For how many ordered pairs (x, y) of integers is it true that $0 < x < y < 1000000$ and that the arithmetic mean of x and y is exactly 2 more than the geometric mean of x and y ?

Compute the positive integer $1 \le k \le 50$ such that $\binom{50}{k}\binom{100}{k}$ is maximized.

Let P be the solution to this problem. If a point is chosen randomly from within the unit square centered at $(0.5, 0.5)$ with sides parallel to the coodinate axes, what is the probability that both of the chosen point's coordinates are greater than P ?

Define the odd factorial $o(n!) = 1 \cdot 3 \cdot 5 \cdot \ldots \cdot n$, where n is a positive odd integer. The largest power of 5 that divides into $o(2019!)$ can be expressed as 5^x , where x is a positive integer. Find x .

Compute the product of all positive integers n for which $1! + 2! + \cdots + n!$ is a perfect square.

Find the sum of the exponents of the prime factorization of $\sum_{m=0}^{18} (2m+1)^3$

Alice, Bob, Charlie, and Daniel are sharing 16 indistinguishable cookies. There exist societal norms that render it infeasible to divide up the cookies evenly. These norms further dictate that every person must receive at least one cookie, and (in the spirit of adultered socialism) no person may have more cookies than the other three people have combined. Lastly, no cookie may be left untouched, as wasting food is a sin. Given these constraints, in how many ways can the cookies be divided among the 4 (distinguishable) people?

Triangle ABC has side lengths $AB = 700, AC = 900, BC = 1007$ and incenter (intersection of angle bisectors) I . Suppose the circle with center I that passes through A intersects side BC at points X and Y . Compute XY .

Suppose there is a triangle ABC such that $AB = 13$, $BC = 14$, and $CA = 15$. Rotate this figure 180 degrees about its centroid. Find the area of the intersection of the 2 triangles.

SEASON 6

Suppose $a_1, a_2, \cdots, a_{100}$ are real numbers that lie in the interval $[-1, \infty)$. Suppose $a_1^2 + a_2^2 + \cdots + a_{100}^2 = 250$. What is the least possible value of $a_1^3 + a_2^3 + \cdots + a_{100}^3$?

Find the number of positive integers a such that there exists a positive integer b satisfying $a^3 < 5ab < a^3 + 100$.

How many ways can Joe get from $(0,0)$ to $(10,10)$ while moving only 1 unit either North or East at a time given that Bob the Builder is building a house and requires the square with vertices $(4,4)$, $(4,6)$, $(6,4)$, and $(6,6)$ to be cordoned off (including the square's boundary)?

Suppose we have complex number w such that $w^{23} = 1$ .The sum $\sum_{n=0}^{22} \frac{1}{1 + w^n + w^{2n}}$ can be expressed as $\frac{a}{b}$. Find $a + b$.

W ≠ 1

Suppose $\triangle ABC$ has a circumradius of 15 , O is the circumcenter, and G is the centroid. Furthermore, let M be the midpoint of BC . Given that $BC = 18$ and $\angle MOA = 150°$, what is the area of $\triangle OMG$?

Compute the smallest positive integer m such that $1 \cdot 3^1 + 2 \cdot 3^2 + 3 \cdot 3^3 + \cdots + m \cdot 3^m$ exceeds 3^{2007} .

An unfair, 2 sided coin has a probability S of coming up heads in any toss. Let the coin be flipped twice, and let $P(S)$ be the probability that heads comes up exactly once. Albernaut informs you of the value $P(S_1)$, allowing you to uniquely determine S_1 . Find $\frac{S_1}{P(\frac{1}{3})}$

Find the number of ordered triples of integers (a, b, c) such that $a^2 + b^2 - 8c = 6$.

You are given 30 distinct real numbers. These numbers are put in random order to form a list of 30 numbers. From here, the first and second numbers are compared, and if the second number is greater than the first, the numbers are swapped. Then, the second number is compared with the third number, and the pattern continues on. What is the probability that a list is formed such that the 21st element ends up in the 26th spot after the aforementioned process is performed on the list?

A and B are the roots of equation $x^2 - yx + 2 = 0$. If $B + \frac{1}{a}$ and $A + \frac{1}{B}$ are the roots of equation $x^2 - px + q = 0$, what is q ?

Define S to be the set of integers formed by $2^a + 2^b + 2^c$ where $a, b,$ and c are pairwise distinct positive integers. Find the sum of the digits of the 100^{th} smallest element of S .

A positive integer Y is called a suave number if the absolute difference between every pair of consecutive digits is greater than 6 when Y is written in base 10. How many eight digit numbers are suave?

A positive integer Y is called a suave number if the absolute difference between every pair of consecutive digits is greater than 6 when Y is written in base 10. How many eight digit numbers are suave?

The function f is defined by $f(n)$ being the n^{th} positive integer that is not a perfect square. Given that $\underbrace{f(f(\cdots f}_{2019\ f\text{'s}}(n)\cdots)) = 2020^2 + 1$ for some positive integer n , compute n .

Pranav has two bags, each with 6 marbles in them. Bag 1 contains 4 white marbles and 2 red marbles. Bag 2 contains 3 black marbles and 3 red marbles. Pranav randomly selects a bag, then randomly selects a marble from within. This procedure is repeated, without returning the marbles to the bags. The probability that the first ball drawn is red given that the second ball drawn is black can be expressed as $\frac{m}{n}$. Find $m + n$.

How many 3 -digit base- 5 numbers are divisible by 7 such that if the ordering of digits was instead an ordering of base 10 digits, the number would still be divisible by 7 ? ($ABC5$ is divisible by 7 , $ABC10$ is divisible by 7 , same three digits both times)

Jim and John are riding their bikes. Jim starts biking from the ice cream shop at 10 miles per hour in a straight-line path that forms a 60° angle with the horizontal. John starts biking at the same time as Jim 50 miles due east from the ice cream shop at 15 miles per hour, and takes a straight-line path such that he can meet Jim in the fastest time possible. The number of hours it takes for Jim and John to meet can be expressed as $a\sqrt{b}-c$, where a, b, c are positive integers and b is square-free. Find $a+b+c$.

What is the probability of not flipping consecutive tails when flipping 8 coins?

Let r_1, r_2 be the roots of the polynomial $x^2-7x+13$. Compute $r_1^4+r_2^4$.

L is a list of not necessarily distinct positive integers in which the number 68 appears at least once. The average of the numbers in L is 56 . If one of the (potentially but not necessarily numerous) " 68 "s is removed, the average of the remaining numbers is 55 . What is the largest number that can appear in L ?

Define S as the sum of all positive base ten integers of the form xyz such that xyz , yzx , and zxy form a geometric progression. Find S mod 23 .

In which row of pascal's triangle do three consecutive entries occur in the ratio 3 : 4 : 5 ?

There exists a cyclic quadrilateral $ABCD$ such that $AB=35$, $BC=75$, $CD=100$, and $DA=120$. Find the length of AC .

There exists a complex number z with $|z|=2019$ and real number $c>1$ such that z, cz, and z^2 form an equilateral triangle in the complex plane. c can be written in the form $\frac{w+\sqrt{x}}{y}$ where w, x, y are positive integers and x is not divisible by the square of any prime. Compute $w+x+y$.

How many positive integers k are there such that $100<k\leq 10000$ and $\lfloor\sqrt{k-100}\rfloor$ divides k ? Note that $\lfloor\rfloor$ denotes the floor function.

Let set A be a 90 -element subset of $\{1,2,3,\ldots,100\}$, and let S be the sum of the elements of A . Find the number of possible values of S .

Find the number of 5-digit positive integers $\overline{abcde}$ such that $10a+b+10b+c+10c+d+10d+e=100$.

Suppose a, b, c are complex numbers such that $a^2 + 5b = 10a, b^2 + 5c = 10b, c^2 + 5a = 10c$. Compute the sum of all possible values of c .

10 positive integers are placed around a circle. Each integer is one more than the greatest common divisor of its (two) neighbors. What is the sum of the 10 numbers?

$\sqrt{170} = a + \frac{1}{b + \frac{1}{b + \ldots}}$ where a and b are positive integers. What is $a + b$?

Consider the sequence $1, 3, 4, 7, 8, 9, 13, 14, 15, 16, \cdots$ where you list the first positive integer, then skip the next positive integer, list the next 2 positive integers, then skip the next 2 positive integers, list the next 3 positive integers, then skip the next 3 positive integers and so forth. What is the 2019th term of this sequence?

Let a, b, c be positive integers such that $\operatorname{lcm}(a, b) = 2^3 \cdot 7^2, \operatorname{lcm}(b, c) = 2^3 \cdot 7^3, \operatorname{lcm}(a, c) = 2^2 \cdot 7^3$. How many ordered triples of positive integers (a, b, c) exist such that the previously stated constraints are satisfied?

Compute the number of positive integers $k \leq 2019$ such that there exists an integer n for which $\dfrac{n+k}{n-k}$ is an odd perfect square.

SEASON 7

Suppose a quadratic polynomial $p(x)$ with integer coefficients has $p(41) = 42$. Given that there exists integers $i, j > 41$ such that $p(i) = 13$ and $p(j) = 73$, what is the value of $p(1)$?

Find the number of ordered pairs of integers (a, b) with $-2019 \leq a, b \leq 2019$ such that the equation $m^3 + n^3 = a + 3bmn$ has infinitely many integer solutions (m, n) .

The problems from 8/19 - 8/23 could not be retrieved.

Find the smallest positive integer a so that there exists at least 3 different ordered pairs of positive integers (m, n) so that $m^2 - n^2 = a$.

The problems from 8/25-8/31 could not be retrieved.

Rectangle $ABCD$ is such that $AB = 5$ and $BC = 6$. Point E lies on diagonal AC and point F is such that EF is parallel to AB and the distance from F to BC is $\frac{3}{2}$ the distance from E to BC. Given that $AFEB$ forms an isosceles trapezoid, the length of BF can be expressed as $\frac{a\sqrt{b}}{c}$ where b is not divisible by the square of any prime and $\gcd(a, c) = 1$. Compute $a - b + c$.

Define the binary operation $a \heartsuit b = \dfrac{max(a,b)}{min(a,b)}$ on the positive reals. The quantity $\prod_{i=1}^{2020} \dfrac{M_i}{N_i}$ where M_i is the maximum and N_i is the minimum value of $a_1 \heartsuit a_2 \heartsuit \ldots \heartsuit a_i$ (evaluated from left to right) where the a_j's are a permutation of $2^0, 2^1, 2^2, \ldots, 2^{i-1}$ can be expressed as 2^k. Compute $k \mod 1000$. ×
(Proposed by Albert Wang)

Let $\triangle ABC$ be a triangle with $AB = 37, BC = 684, AC = 685$. Define M as the midpoint of BC. Define X and Y as trisection points along AC. Let D and E be the intersection of BX and BY with AM respectively. If the area of quadrilateral $XYED$ can be expressed as $\frac{m}{n}$ for $\gcd(m,n) = 1$, compute $m \pmod{1000}$.

Triangle ABC satisfies $AB = 8, BC = 7, CA = 5$. A circle ω_B centered at O_B is tangent to line AB at B and passes through C. Circle ω_C, centered at O_C, passes through B and is tangent to line CA at C. If the circumcenter of $\triangle ABC$ is at O, then the quantity $OO_B + OO_C$ can be expressed as $\frac{m\sqrt{r}}{n}$ for squarefree r and $\gcd(m,n) = 1$. Find $m + n + r$.
(Proposed by Albert Wang)

Suppose x, y, w, z are real numbers such that $x + y + w + z = 20$ and $xy + yw + wz + zx = 16$. Compute the maximum possible value of $xyw + ywz + wzx + zxy$.

Evaluate $\displaystyle\sum_{a=1}^{\infty} \frac{2a-1}{3^a}$

Suppose p is a polynomial with integer coefficients such that $p(0) \geq 0$ and $p(-34) = 2014$. Furthermore, assume $p(35)p(21)p(15)$ has a remainder of 10 upon division by 105. What is the minimum possible value of $p(0)$?

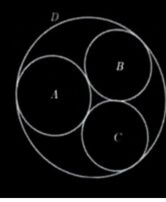
How many 9-letter strings of the letters A and B do not contain the string $ABBA$?

Let $g(x)$ be a function such that the values $\{g(1), g(2), g(3) \ldots g(1000)\}$ directly map to a permutation of $\{1, 2, 3 \ldots 1000\}$, such that the function satisfies $g(10) > g(100)$ and $g(20) > g(200)$. Let the probability that $g(10) > g(200) > g(300)$ be p. p can be expressed as $\frac{a}{b}$, where $\gcd(a,b) = 1$. Find $a + b$.

A large group of people are standing in a line. Starting with the first person in line, each person will draw from a deck of 2019 cards, numbered 1 to 2019 with replacement. The first person to draw the same number as someone before him/her wins. The person with the highest probability of winning is standing n^{th} in line. Find n.

Suppose triangle ABC has incircle Γ (the circle tangent to the 3 sides of the triangle) and has circumcenter O. Given that O lies on the circumference of Γ and $\angle A = 60°$, $\tan C \tan B$ can be written as $x + \sqrt{y}$ where x and y are positive integers. What is xy?

Circles A, B, and C are externally tangent to each other and internally tangent to circle D. Suppose B and C are congruent with radius 8.
Given that A passes through the center of D,
find the radius of A.



Compute the sum of all positive integers k for which the decimal expansion of $\frac{2k+1}{k(k-1)}$ terminates.
(Proposed by Evan Chen)

Let p and q be positive integers $1 \le p, q \le 100$ such that $p + q | pq + 1$ and $p + q$ is prime. Find the number of **unordered** pairs $\{p, q\}$ that satisfy the aforementioned constraints. (The notation $x|y$ means that x evenly divides into y, or $y \equiv 0 \pmod{x}$).

An equilateral triangle $\triangle ABC$ of side-length 2 is drawn. For side AB, a square is drawn such that it has side AB and shares its interior with $\triangle ABC$. This process is repeated for sides BC and AC. Finally, an equilateral triangle is drawn, containing all three of these squares. Let the smallest possible perimeter this triangle can have be equal to $a\sqrt{b} + c$ with integers a, b, c, and square-free b. Find $a + b + c$.

SEASON 8

Triangle $A_0B_0C_0$ has $A_0B_0 = 20$, $B_0C_0 = 21$, and $m_0B_0C_0 = 90°$. For all positive integers n, the segment B_nC_n is parallel to B_0C_0 and tangent to the incircle of triangle $A_0B_{n-1}C_{n-1}$, such that $B_0C_0 > B_1C_1 > \cdots > B_nC_n$. The sum of the lengths of $B_{n+1}C_n$ for all positive integers n is equal to $a\sqrt{b}$ for integer a and square-free b. Find $2a + b$.
(Proposed by @Cylence)

Given that ABC is an isosceles right triangle, let the operator $\gamma(n)$ be such that an altitude is drawn from every right angle in ABC, n times. If there are k squares completely inside ABC for $\gamma(2019)$, find $2k \pmod{32}$.
(Proposed by @Γishabh β)

Let line α be a number line marked with all the integers. There are 3 balls at each of the points $6, 7, 8, 9, 10, 11, 12$ on line α. Every second, **one** ball moves either 1 unit to the left or 1 unit to the right of its current position. What is the minimum amount of seconds that will pass before no two balls are on the same coordinate on line α?

Suppose L is the set $\{2, 5, 8, 11, 14, 17, 20, \ldots\}$. Given that it's possible to choose n distinct numbers from L such that the sum of their reciprocals is 1, compute the smallest possible value of n.

The fraction $\frac{2}{9}$ can be expressed as a repeating decimal in base 11. Find the sum of the first 30 digits to the right of the decimal point.

Let Q be a polynomial such that $Q(n) = \binom{2018}{n}$ over the interval $0 \le n \le 2018$ where n is an integer, and $\deg Q \le 2018$. Find the largest positive integer value of m such that 2^m evenly divides $Q(2020)$.

Consider the function $g(a, b) = a^{\log_2 b}$ for $a, b > 0$. The sum of the solutions of the equation $4096g(g(z, z), z) = z^{13}$ can be expressed as $\frac{m}{n}$ where m and n are relatively prime positive integers. Find $m + n$.

If 11111_{10} is a perfect square when expressed in base b, find b.

There are 6 people sitting around a circular table. In order, the seats are labeled $1, 2, 3, 4, 5, 6$. Each second you move to the seat directly opposite you or the two seats adjacent to the opposite seat with equal probability. If you are in seat 2 at 0 seconds, the expected number of seconds until you are in seat 1 can be expressed as $\frac{m}{n}$ where m and n are relatively prime. Find m+n.
-Proposed by Jonathan Pei (@RadiantCheddar)

POTDBot is very SMOL and thus fits into the square with vertices $(0, 0), (1, 0), (0, 1)$, and $(1, 1)$. However, it is also very FAST and will move 1 block in any of the cardinal directions (N,S,W,E) every 2 seconds. Lastly, it is very DUMB and so makes moves at random, never staying still. How many different squares can POTDBot land up on after 200 seconds pass?

Call the number of ways Akshar, Pranav, Kenan, Noah, Sushrit, Srijan, Adi, and Eli can recieve 9 distinct gifts x. Call the number of ways 8 Hari's can receive 9 distinct gifts y. Find $x + y$.

Consider the set of quintuples of positive integers $1, 2, 3, 4, 5$. We say the quintuple (a, b, c, d, e) (numbers can be repeated, so $(1, 1, 2, 2, 3)$ can be a quintuple here). Call a quintuple **good** if any three numbers chosen from that quintuple do **not** form an arithmetic sequence. Find the number of good quintuples.

Suppose x, y, z are nonzero real numbers that satisfy $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + \frac{1}{xyz} = 1$ and $x + y + z = 12$. Find the largest possible value of $xyz - x - 2y + 3z$.

Compute the remainder when

$$2\left(1 \cdot \binom{2019}{1} + 2 \cdot \binom{2019}{2} + 3 \cdot \binom{2019}{3} + \cdots + 2019 \cdot \binom{2019}{2019}\right)$$

is divided by 25.

Find the smallest positive integer n for which

$$\gcd(n^3 + 3n + 1, 7n^3 + 18n^2 - n - 2) > 1.$$

The probability that a randomly selected positive integer divisor, d, of $12!$ satisfies $d \equiv 1 \pmod 3$ can be expressed as $\frac{m}{n}$, where m and n are positive integers such that $\gcd(m, n) = 1$. Find $m + n$.

Let P be the sum of all integers A such that $x^2 + Ax + 2008A$ has integer root(s). Find$|P|$

There are positive integers such that $y^2 = x^2 + 24x + 460$. Find $30y - 30x$.

10/08 could not be retrieved.

Find the maximum attainable value of $a + b + c$ that is less than 200 where a, b, c are positive integers that satisfy $a^c + 1 = b^{c+1}$ and $\gcd(a, c + 1) = 1$.

8 coins are arranged in a circle, all facing heads up. Define a move as flipping over 2 adjacent coins. Find the number of sequences of 6 moves that result in the coins alternating between heads up and tails up.

Let N be the sum of all integers that can be written in the form $2^x \cdot 3^y \cdot 5^z$ where x, y, z are positive integers so that $x + y + z = 10$. What is the remainder when N is divided by 1001?

Given that $a^2 + b^2 = 3b + 1$ for real numbers a and b, the maximum value of $a + b$ is m. m can be expressed as $\frac{x + \sqrt{y}}{z}$ for positive integers x, y, z and square-free y. Find $x + y + z$.

Find the number of positive integers x less than 100 such that

$$3^x + 5^x + 7^x + 11^x + 13^x + 17^x + 19^x$$

is prime

Mr. Lyttle's 3rd Period class is having a Dual Tournament! The bracket can be seen at https://challonge.com/8etx1r4i . Rishabh and Rohit would like to face each other. Call the probability of this happening $\frac{m}{n}$, when m and n are relatively prime positive integers. Find m.

The greatest possible radius of a circle that passes through the points $(1, 2)$ and $(4, 5)$ such that its interior is contained in the first quadrant of the coordinate plane can be expressed as $a + b\sqrt{c}$. Compute $a + b + c$.

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This morning, Pranav asked PoTD Bot$^{\text{TM}}$ to display the numbers 1-10, in order. However, since PoTD Bot$^{\text{TM}}$ is a bot, it forgot how to count properly. Instead, it displayed a permutation of the numbers $\{1, 2, \ldots, 10\}$ on the screen in a manner that fit the following: The leftmost number was 1, The rightmost number was 10, and exactly one number (excluding 1 or 10) is less than both the numbers to its immediate left and right. How many possible ways could PoTD Bot$^{\text{TM}}$ have displayed the numbers?

Let a, b, c be the zeroes of $y = 9x^3 + 69x + 2019$ Find $(a + b)^3 + (b + c)^3 + (c + a)^3$.

SEASON 9

A triangle $\triangle ABC$ has $\angle A = 60^\circ$, $BC = 69$ and incircle ω. When ω is inverted about Ω, the circumcircle of $\triangle ABC$, to produce circle ω^*, the orthocenter H lies on ω^*. The value $\tan\left(\frac{\angle B}{2}\right) \cdot \tan\left(\frac{\angle C}{2}\right)$ can be expressed in the form $\frac{n - \sqrt{m}}{k}$ for squarefree m and $\gcd(n, k) = 1$. Find $n + m + k$. (Note: Inversion about a circle with radius r and center O takes any point P to the point P^* on ray OP for $OP \cdot OP^* = r^2$) **Proposed by Albert Wang**

Suppose ABC is an equilateral triangle. Construct points O_n on AB such that $\frac{BO_n}{AB} = \frac{2}{4^n}$ and points P_m on AC such that $\frac{CP_m}{AC} = \frac{1}{4^m}$. The sum of the areas of triangles $O_k O_{k+1} P_k$ divided by the area of $\triangle ABC$ where k spans over the positive integers can be expressed as $\frac{x}{y}$ where x and y are relatively prime positive integers. Compute $x + y$.
(Proposed by Harry Chen)

Let $f(x) = \sin(\cos(x) + a)$ and $g(x) = \cos(\sin(x))$ where $0 \leq a < 2\pi$. What is the sum of all values of a such that there exists only 2 solutions to the equation $f(n) = g(n)$ where $0 \leq n < 2\pi$. Convert your final answer from radians to degrees.
(Proposed by Harry Chen)

Find the remainder when $\sum_{n=0}^{333} \sum_{k=3n}^{999} \binom{k}{3n}$ is divided by 70.
(Proposed by Jonathan Pei (@RadiantCheddar))

Find the maximum positive integer a such that for any real number b we can write $\sin^a b + \cos^a b \geq \frac{1}{a}$.

Compute the number of integers n between 1 and 1336 inclusive such that $\binom{1337}{n+1}\binom{1337}{n-1}$ is divisible by $\binom{1337}{n}$.

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f(a,b) means f(ab)=f(a*b)
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SEASON 10

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